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FREE VIBRATION ANALYSIS OF CSCS &CSSS RECTANGULAR PLATE BY SPLIT-DEFLECTION METHOD

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***Abstract*—*This work presents free vibration analysis of rectangular plate by split-deflection method. In this method, the deflection was split into x and y components of deflection. That is the deflection of the rectangular plate was taken as the product of these two components. Having made this assumption, the study went ahead to formulate total potential energy functional from principles of theory of elasticity based on work-error approach. This energy function was minimized by direct variation and equation for resonating frequency was obtained. Two illustrative examples were used to test this method. They are (i) plate with edges 1 & 3 clamped and edges 2 & 4 simple supported (CSCS), and (ii) plate with edge 1 clamped and the other three edges simple supported (CSSS). The first and second examples used polynomial function for both x and y components of deflection. Fundamental resonating frequencies (in non-dimensional forms) of the two plates for aspect ratios ranging from 1.0 to 2.0 (at increment of 0.1) were determined and compared with the values from previous study. From the comparison, it was observed that the maximum percentage difference of -0.07 was recorded for the first example at aspect ratios of 2.0 with non-dimensional resonating frequency of 13.72. For the second example, there was no difference between the present and previous studies.*

***Keywords*—Deflection, Resonating-frequency, Split-deflection, Work-error, Energy function,Non-dimensional, Polynomial function.

INTRODUCTION

Most scholarly works on classical plate theory (CPT) analysis of rectangular plates rely on single orthogonal function (Hutchinson, 1992; Ibearugbulem, 2014). Obviously, one can affirm that all energy methods in use are based on single orthogonal deflection function and none has used a deflection function that is typically separated into two independent distinct functions ($w = w_x * w_y$). Some energy methods used for free vibration analysis of rectangular plates include Raleigh, Raleigh-Ritz, Ritz, Galerkin, minimum energy potential, work-error etc (Ugural, 1999, Eduard and Kranthammer, 2001, and Ibearugbulem et al.,2014).The deflection (displacement normal to the plane of the plate) is a single orthogonal function , w. This is apparent in the energy function for Raleigh, Raleigh-Ritz and Ritz. Typical energy function is (Ibeaugbulem et al., 2014):

$$\Pi = \frac{D}{2}\int_0^a\int_0^b\left(\left[\frac{\partial^2 w}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 w}{\partial x\partial y}\right]^2 + \left[\frac{\partial^2 w}{\partial y^2}\right]^2\right)\partial x\partial y$$

$$-\frac{m\,.\,\lambda^2}{2}\int_0^a\int_0^b w^2\,\partial x\partial y$$

The use of single orthogonal deflection function is also seen in Galerkinand work-error methods. Typical work-error function is (Ibearugbulem et al., 2014):

$$\Pi = \frac{D}{2}\int_0^a\int_0^b\left(\frac{\partial^4 w}{\partial x^4}w + 2\frac{\partial^4 w}{\partial x^2\partial y^2}w + \frac{\partial^4 w}{\partial y^4}w\right)\partial x\partial y$$

$$-\frac{m\,.\,\lambda^2}{2}\int_0^a\int_0^b w^2\,\partial x\partial y$$

In this present study$w_x$and $w_y$are polynomials functions.The advantage of this method is that deflection is split into two for in-depth study. Also it presents alternative reliable method of free vibration analysis to engineers. Also analyst who may difficulty in obtaining orthogonal function for a plate of a particular boundary condition will have this method as a way out. In this case, the analyst who may have easy access to deflection equations for beams of any boundary condition can find the proposed method quite useful and handy.

ASSUMPTIONS

1.Basic- The assumption here is that the general deflection, w is split into w_x and w_y. That is the split-deflection function is given as:

$$w = w_x * w_y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.1$$

Where the w_x and w_y components of the deflection are defined as:

$$w_x = \sqrt{A} * h_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..2$$

$$w_y = \sqrt{A} * h_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3$$

Substituting equations (2) and (3) into equation (1) gives:

$$w = A\,h_1 h_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..4$$

PRINCIPLES OF THEORY OF ELASTICITY

2. In-Plane Displacements- From the assumption that vertical shear strains equal to zero for classical plate, and making use of split-deflection method, we have:

$$u = -z\frac{dw}{dx} = -z\frac{dw_x}{dx}w_y \ldots\ldots\ldots\ldots\ldots.5$$

$$v = -z\frac{dw}{dy} = -z\frac{dw_y}{dy}w_x \ldots\ldots\ldots\ldots\ldots 6$$

3.Strain-Deflection Relationship- Using equations (5) and (6), the three in-plane strains of CPTare obtained as:

$$\varepsilon_x = \frac{du}{dx} = -z\frac{d^2w_x}{dx^2}w_y \ldots\ldots\ldots\ldots\ldots\ldots.7$$

$$\varepsilon_y = \frac{dv}{dy} = -z\frac{d^2w_y}{dy^2}w_x \ldots\ldots\ldots\ldots\ldots\ldots 8$$

$$\gamma_{xy} = \frac{du}{dy} + \frac{dv}{dx} = -2z\frac{dw_x}{dx}\frac{dw_y}{dy} \ldots\ldots 9$$

4. Stress – Strain Relationship-TheCPT constitutive equations for plane stress plate are given as:

$$\sigma_x = \frac{E}{1-\mu^2}[\varepsilon_x + \mu\varepsilon_y] \quad \ldots\ldots\ldots\ldots\ldots\ldots 10$$

$$\sigma_y = \frac{E}{1-\mu^2}[\mu\varepsilon_x + \varepsilon_y] \quad \ldots\ldots\ldots\ldots\ldots\ldots 11$$

$$\tau_{xy} = \frac{E(1-\mu)}{2(1-\mu^2)}\gamma_{xy} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 12.$$

DERIVATION OF FREE-VIBRATION LOAD EQUATION APPLYING SPLIT-DEFLECTION METHOD.

5. Stress – Deflection Relationship- Substituting equations (7), (8) and (9) into equations (10), (11) and (12) accordingly gives the split-deflection stress-deflection equation as:

$$\sigma_x = \frac{-Ez}{1-\mu^2}\left[\frac{d^2w_x}{dx^2}w_y + \mu\frac{d^2w_y}{dy^2}w_x\right] \ldots\ldots 13$$

$$\sigma_y = \frac{-Ez}{1-\mu^2}\left[\mu\frac{d^2w_x}{dx^2}w_y + \frac{d^2w_y}{dy^2}w_x\right] \ldots\ldots\ldots 14$$

$$\tau_{xy} = \frac{-Ez(1-\mu)}{(1-\mu^2)}\frac{dw_x}{dx}\frac{dw_y}{dy} \ldots\ldots\ldots\ldots\ldots 15$$

6. Total Potential Energy -Total potential energy is mathematically defined as: $\prod = U - V$

U = Strain energy or internal work

V = External work.

The strain energy is defined as:

$$U = \frac{1}{2}\int_x \int_y \left[\int_{-\frac{t}{2}}^{\frac{t}{2}} [\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy} \Upsilon_{xy}] dz\right] dx\, dy \ldots\ldots\ldots\ldots 16$$

For pure bending analysis, the external workdone by free vibration is given as:

$$V = \int_x \int_y \frac{m.\lambda^2}{2} w_x{}^2 . w_y{}^2 dx\, dy$$

That is

$$V = \frac{m.\lambda^2}{2}\int_x w_x{}^2\, dx \int_y w_y{}^2\ dy\ \ldots\ldots\ldots.17$$

Substituting equations (7), (8), (9),(13), (14)and (15) into equation (16) gives strain energy – deflection relationship as:

$$U = \frac{D}{2}\int_x \int_y \left[\left(\frac{d^2 w_x}{dx^2}\right)^2 + 2\left(\frac{dw_x}{dx}\right)^2\left(\frac{dw_y}{dy}\right)^2 + \left(\frac{d^2 w_y}{dy^2}\right)^2 w_x{}^2\right] dx\, dy$$

In work-error approach, the strain energy becomes:

$$U = \frac{D}{2}\left[\int_x \frac{d^4 w_x}{dx^4} w_x\, dx \int_y w_y{}^2\, dy\right]$$

$$+\frac{2D}{2}\left[\int_x \frac{d^2 w_x}{dx^2} w_x\, dx \int_y \frac{d^2 w_y}{dy^2} w_y\, dy\right]$$

$$+\frac{D}{2}\left[\int_x w_x{}^2\, dx \int_y \frac{d^4 w_y}{dy^4} w_y\, dy\right] \ldots\ldots\ldots\ldots\ 18$$

Subtracting equation (17) from Equation (18) gives the total potential energy function as:

$$\Pi = \frac{D}{2}\left[\int_x \frac{d^4 w_x}{dx^4} w_x\, dx \int_y w_y{}^2\, dy\right]$$

$$+\frac{2D}{2}\left[\int_x \frac{d^2 w_x}{dx^2} w_x\, dx \int_y \frac{d^2 w_y}{dy^2} w_y\, dy\right]$$

$$+\frac{D}{2}\left[\int_x w_x{}^2\, dx \int_y \frac{d^4 w_y}{dy^4} w_y\, dy\right]$$

$$-\frac{m.\lambda^2}{2}\int_x w_x{}^2 dx \int_y w_y{}^2\, dy \ldots..19$$

Substituting equations (2) and (3) into equation (19) gives:

$$\Pi = \frac{A^2 D}{2}\left[\int_x \frac{d^4 h_1}{dx^4} h_1\, dx \int_y h_2{}^2\, dy\right]$$

$$+\frac{2A^2D}{2}\left[\int_x \frac{d^2h_1}{dx^2}h_1\,dx\int_y \frac{d^2h_2}{dy^2}h_2\,dy\right]$$

$$+\frac{A^2D}{2}\left[\int_x h_1{}^2\,dx\int_y \frac{d^4h_2}{dy^4}h_2\,dy\right]$$

$$-\frac{m.\lambda^2}{2}A^2\int_x h_1{}^2dx\int_y h_2{}^2\,dy....20$$

Now, equation (20) can be written in non- dimensional axes R and Q.

$$x = aR \quad \text{...} \quad 21$$

$$y = bQ \quad \text{...} \quad 22$$

$$P = \frac{b}{a} \quad \text{...} \quad 23$$

Where a & b are the plate lengths in x and y axes respectively and p is the long span-short span aspect ratio.

Substituting equations (21), (22) and (23) into equation (20) gives:

$$\Pi = \frac{abA^2D}{2a^4}\left[\int_0^1 \frac{d^4h_1}{dR^4}h_1dR\int_0^1 h_2{}^2\,dQ\right]$$

$$+2\frac{abA^2D}{2a^4P^2}\left[\int_0^1 \frac{d^2h_1}{dR^2}h_1dR\int_0^1 \frac{d^2h_2}{dQ^2}h_2\,dQ\right]$$

$$+\frac{abA^2D}{2a^4P^4}\left[\int_0^1 h_1{}^2\,dR\int_0^1 \frac{d^4h_2}{dQ^4}h_2\,dQ\right]$$

$$-\frac{m.\lambda^2}{2}A^2ab\int_0^1 h_1{}^2dR\int_0^1 h_2{}^2\,dQ \quad ...24$$

7. Direct Variation of Total Potential Energy-Equation (24) is differentiated with respect to the deflection coefficient, A and the result is:

$$\frac{d\Pi}{dA} = \frac{AD}{a^4}\left[\int_0^1 \frac{d^4h_1}{dR^4}h_1dR\int_0^1 h_2{}^2\,dQ\right]$$

$$+2\frac{AD}{a^4P^2}\left[\int_0^1 \frac{d^2h_1}{dR^2}h_1dR\int_0^1 \frac{d^2h_2}{dQ^2}h_2\,dQ\right]$$

$$+\frac{AD}{a^4P^4}\left[\int_0^1 h_1{}^2\,dR\int_0^1 \frac{d^4h_2}{dQ^4}h_2\,dQ\right]$$

$$-m.\lambda^2A\int_0^1 h_1{}^2dR\int_0^1 h_2{}^2\,dQ = 0$$

That is $\frac{d\Pi}{dA} =$

$$\frac{D}{a^4}\left[\int_0^1 \frac{d^4h_1}{dR^4} h_1 dR \int_0^1 {h_2}^2 \, dQ\right]$$
$$+2\frac{D}{a^4P^2}\left[\int_0^1 \frac{d^2h_1}{dR^2} h_1 dR \int_0^1 \frac{d^2h_2}{dQ^2} h_2 \, dQ\right]$$
$$+\frac{D}{a^4P^4}\left[\int_0^1 {h_1}^2 \, dR \int_0^1 \frac{d^4h_2}{dQ^4} h_2 \, dQ\right]$$
$$-m.\lambda^2 \int_0^1 {h_1}^2 dR \int_0^1 {h_2}^2 \, dQ \;\ldots 25$$

This equation (25) is the direct governing equation of rectangular plate under free vibration using work-error approach from this present method. Re-arranging equation (25) and making resonating frequency, λ the subject of the equation gives:

$$\lambda^2 = \left(\frac{k_x + 2\frac{k_{xy}}{P^2} + \frac{k_y}{P^4}}{k_\lambda}\right) * \frac{D}{ma^4} \;\ldots\ldots\ldots\ldots 26$$

Where

$$k_x = \int_0^1 \frac{d^4h_1}{dR^4} h_1 dR \int_0^1 {h_2}^2 \, dQ \;\ldots\ldots\ldots\ldots 27$$

$$k_{xy} = \int_0^1 \frac{d^2h_1}{dR^2} h_1 dR \int_0^1 \frac{d^2h_2}{dQ^2} h_2 \, dQ \;\ldots\ldots 28$$

$$k_y = \int_0^1 {h_1}^2 \, dR \int_0^1 \frac{d^4h_2}{dQ^4} h_2 \, dQ \;\ldots\ldots\ldots 29$$

$$k_\lambda = \int_0^1 {h_1}^2 dR \int_0^1 {h_2}^2 \, dQ \;\ldots\ldots\ldots\ldots 30$$

APPLICATION

Analysis of a classical rectangular thin isotropic plate with:

i.Edges 1 & 3clamped and edges 2 & 4 simply supported using polynomial function for both w_x and w_y.

ii, Edge 1 clamped and the other three edges simple supported using also polynomial function for both w_x and w_y

In the derivation of the particular resonating split-deflection polynomial plate equations, the following deflection expression, w, in expanded form was used.

$$w = (\alpha_o + \alpha_1 R + \alpha_2 R^2 + \alpha_3 R^3 + \alpha_4 R^4) * (\beta_o + \beta_1 Q + \beta_2 Q^2 + \beta_3 Q^3 + \beta_4 Q^4) \ldots\ldots\ldots 3$$
1

Recall w = $w_{(x)} * w_{(y)}$

$$\therefore w(x) = (\alpha_o + \alpha_1 R + \alpha_2 R^2 + \alpha_3 R^3 + \alpha_4 R^4) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 32$$

$$w(y) = (\beta_o + \beta_1 Q + \beta_2 Q^2 + \beta_3 Q^3 + \beta_4 Q^4) \ldots\ldots\ldots\ldots\ldots \quad 33$$

1.Rectangular plate with edges 1 & 3 clamped and edges 2 & 4 simple supported (CSCS).

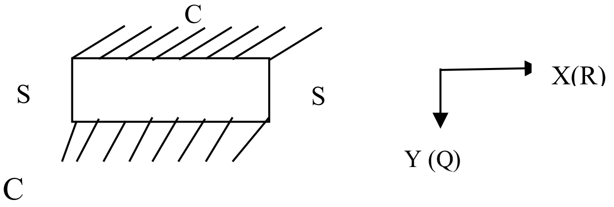

Figure 1: CSCS Rectangular Plate

Along x- direction:

$$w(x) = (\alpha_o + \alpha_1 R + \alpha_2 R^2 + \alpha_3 R^3 + \alpha_4 R^4) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 34$$

$1st\ derivative\ of\ w(x) = w'(x)$

$= (\alpha_1 + 2\alpha_2 R + 3\alpha_3 R^2 + 4\alpha_4 R^3) \ldots\ldots 35$

$2nd\ \ derivative\ of\ w(x) = w''(x)$

$= (2\alpha_2 + 6\alpha_3 R + 12\alpha_4 R^2) \ldots\ldots\ldots\ldots \quad 36$

Boundary conditions:

At,R=0, $w(R = 0)$

$= \alpha_0 + 0 + 0 + 0 + 0. \Rightarrow \alpha_0 = 0$

$w''(R = 0) = 2\alpha_2 + 0 + 0. \Rightarrow 2\alpha_2 = 0, \quad \alpha_2 = 0$

$\therefore \alpha_0 = 0\ ;\ \ \alpha_2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 37$

At R= 1:

$$w(1) = 0 = (\alpha_o + \alpha_1 R + \alpha_2 R^2 + \alpha_3 R^3 + \alpha_4 R^4)$$

$\therefore w(1) = 0 = \alpha_1 + \alpha_3 + \alpha_4 \quad \ldots\ldots\ldots\ldots\ldots 38$

$w''(1) = 0 = 2\alpha_2 + 6\alpha_1 R + 12\alpha_4 R^2$

$\therefore w''(1) = (\alpha_3 = -2\alpha_4) \ldots\ldots\ldots\ldots\ldots\ldots \ 39$

Substitute α_3 in equation (38)

$\therefore \alpha_1 - 2\alpha_4 + \alpha_4 \ = 0$

$\Rightarrow \alpha_1 = 2\alpha_4 - \alpha_4 \Rightarrow \alpha_1 = \alpha_4 \ldots\ldots\ldots 40$

Summary: $\alpha_0 = 0, \alpha_1 = \alpha_4, \alpha_2 = 0, \alpha_3 = -2\alpha_4$

Substitute these constants in equation (34).

$w(x) = 0 + \alpha_4 R + 0 - 2\alpha_4 R^3 + \alpha_4 R^4$

$w(x) = \alpha_4 R - 2\alpha_4 R^3 + \alpha_4 R^4$

$w(x) = \alpha_4 (R - 2R^3 + R^4) \ldots\ldots\ldots\ldots 41$

Considering y- direction:

$w(y) = (\beta_o + \beta_1 Q + \beta_2 Q^2 + \beta_3 Q^3 + \beta_4 Q^4) \ldots\ldots\ldots\ldots 42$

$w'(y) = (\beta_1 + 2\beta_2 Q + 3\beta_3 Q^2 + 4\beta_4 Q^3) \ldots.43$

Boundary conditions:

At Q = 0.

$w(Q = 0) = 0 = \beta_0 + 0 + 0 + 0 + 0. \Rightarrow \beta_0 = 0$

$w'(Q = 0) = 0 = \beta_1 + 0 + 0 + 0. \Rightarrow \beta_1 = 0$

At Q=1.

$w(Q = 1) = 0 = \beta_0 + \beta_1 + \beta_2 + \beta_3 + \beta_4$

$w(Q = 1) = 0 = \beta_2 + \beta_3 + \beta_4 = 0$

$\Rightarrow \beta_2 + \beta_3 = -\beta_4 \ldots\ldots\ldots\ldots 44$

$w'(Q = 1) = \beta_1 + 2\beta_2 + 3\beta_3 + 4\beta_4$

$2\beta_2 + 3\beta_3 - 4\beta_4 \ldots\ldots\ldots\ldots 45$

Multiply Eqn. (44) by 2. And subtract from Eqn. (45).$(2\beta_2 +$

$(2\beta_2 + 2\beta_3 - 2\beta_4) \Rightarrow \beta_3 = -2\beta_4$

Substitute β_3 *in equation* (44)

$\therefore \beta_2 - 2\beta_4 = -\beta_4 . \Rightarrow \beta_2 = \beta_4$

Summary $\beta_0 = 0, \beta_1 = 0, \beta_2 = \beta_4, \beta_3 = -2\beta_4$.

Substitute these constants into equation (42)

$w(y) = 0 + 0 + \beta_4 Q^2 - 2\beta_4 Q^3 + \beta_4 Q^4 = 0 \ldots\ldots\ldots\ldots 46$

$\beta_4 Q^2 - 2\beta_4 Q^3 + \beta_4 Q^4 = 0$

$\beta_4 (Q^2 - 2Q^3 + Q^4) = 0 \ldots\ldots\ldots\ldots 47$

Recall $w = w(x) * w(y) = w(R) * w(Q)$

$w = \alpha_4 (R - 2R^3 + R^4) * \beta_4 (Q^2 - 2Q^3 + Q^4)$

$w = \alpha_4 \beta_4 (R - 2R^3 + R^4) * (Q^2 - 2Q^3 + Q^4)$

Let $\alpha_4 \beta_4 = A$

$w = A(R - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \ldots\ldots 48$

$w_x = \sqrt{A}(R - 2R^3 + R^4) \ldots\ldots\ldots\ldots\ldots\ldots\ldots.49$

$w_Y = \sqrt{A}(Q^2 - 2Q^3) \ldots\ldots + Q^4 \ldots\ldots\ldots\ldots\ldots\ldots 50$

From equations (49) and (50), h_1 and h_2 are:

$h_1 = R - 2R^3 + R^4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 51$

$h_2 = Q^2 - 2Q^3 + Q^4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 52$

2.Rectangular plate with Edge 1 clampedand the other 3 edges simple supported (CSSS).

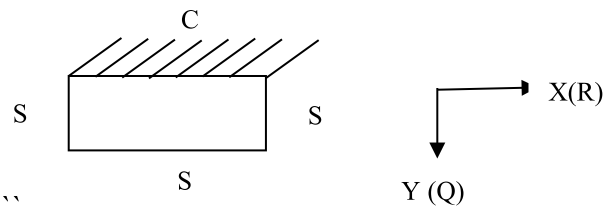

Figure 2.CSSS Rectangular Plate.

In x- direction:

$w(x) = (\alpha_o + \alpha_1 R + \alpha_2 R^2 + \alpha_3 R^3 + \alpha_4 R^4) \ldots\ldots\ldots\ldots\ldots\ldots\ldots.53$

$1st\ derivative\ of\ w(x) = w'(x)$

$= (\alpha_1 + 2\alpha_2 R + 3\alpha_3 R^2 + 4\alpha_4 R^3) \ldots\ldots\ldots 54$

$2nd\ derivative\ of\ w(x) = w''(x)$

$= (2\alpha_2 + 6\alpha_3 R + 12\alpha_4 R^2) \ldots\ldots\ldots\ldots\ldots\ldots 55$

Boundary conditions:

At R=0:, w=0.

$w(R = 0) = \alpha_0 + 0 + 0 + 0 + 0. \Rightarrow \alpha_0 = 0$

$w''(R = 0) = 2\alpha_2 + 0 + 0. \Rightarrow 2\alpha_2 = 0$

$\therefore \alpha_0 = 0;\ \ \alpha_2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 56$

At R= 1:

$w(1) = 0 = (\alpha_o + \alpha_1 R + 0 + \alpha_3 R^3 + \alpha_4 R^4)$

$\therefore w(1) = 0 = \alpha_1 + \alpha_3 + \alpha_4 = 0 \ldots\ldots\ldots 57$

$w''(1) = 0 = 2\alpha_2 + 6\alpha_3 R + 12\alpha_4 R^2$

$\therefore w''(1) = (\alpha_3 = -2\alpha_4)$

Substitute α_3 in equation (57)

$\therefore \alpha_1 - 2\alpha_4 + \alpha_4 \ = 0 \quad \therefore \alpha_1 = 2\alpha_4 - \alpha_4$

$\Rightarrow \alpha_1 = \alpha_4$

Summary: $\alpha_0 = 0, \alpha_1 = \alpha_4, \ \alpha_2 = 0, \ \alpha_3 = -2\alpha_4$

Substitute these constants in equation (53).

$w(x) = 0 + \alpha_4 R + 0 - 2\alpha_4 \ R^3 + \alpha_4 R^4$

$w(x) = \alpha_4 R - 2\alpha_4 \ R^3 + \alpha_4 R^4$

$w(x) = \alpha_4 (R - 2R^3 + R^4) \ldots\ldots\ldots\ldots\ldots .58$

<u>In y – direction</u>

$w(y) = (\beta_o + \beta_1 Q + \beta_2 Q^2 + \beta_3 Q^3 + \beta_4 Q^4) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 59$

1st derivative of w(y) = w'(y)

$= \beta_1 + 2\beta_2 Q + 3\beta_3 Q^2 + 4\beta_4 Q^3$

2nd derivative of w(y) = w″(y)

$= 2\beta_2 + 6\beta_3 Q + 12\beta_4 Q^2$.

<u>Boundary conditions.</u>

At Q = 0,

$w(Q = 0) = \beta_0 + 0 + 0 + 0 + 0. \Rightarrow \beta_0 = 0$

$w'(Q = 0) = \beta_1 + 2\beta_2 Q + 3\beta_3 Q^2 + 4\beta_4 Q^3 \Rightarrow \beta_1 = 0$

At Q =1.

$w(Q = 1) = (\beta_o + \beta_1 Q + \beta_2 Q^2 + \beta_3 Q^3 + \beta_4 Q^4)$

$\therefore w(Q = 1) = \beta_2 + \beta_3 + \beta_4 \ldots\ldots\ldots\ldots\ldots\ldots 60$

$w''(Q = 1) = 2\beta_2 + 6\beta_3 Q + 12\beta_4 Q^2$

$\therefore w(Q = 1) = 2\beta_2 + 6\beta_3 + 12\beta_4$

$\Rightarrow \beta_2 + 2\beta_3 + 6\beta_4 = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots 61$

Subtract $Eqn. (60) from \ Eqn. (61)$.

$(\beta_2 + 2\beta_3 + 6\beta_4) - (\beta_2 + \beta_3 + \beta_4 \ ) = 2\beta_3 + 5\beta_4$

$\Rightarrow \beta_3 = -2.5\beta_4$

$substitute \ \beta_3 \ in \ Eqn. 60$

$\beta_2 - 2.5\beta_4 + \beta_4 \Rightarrow \beta_2 = 1.5\beta_4$

Summary: $\beta_0 = 0, \beta_1 = 0, \beta_2 = 1.5\beta_4, \beta_3 = -2.5\beta_4$.

Substitute these constants into equation (59)

$$w(y) = (0 + 0 + 1.5\beta_4 Q^2 - 2.5\beta_4 Q^3 + \beta_4 Q^4$$

$$w(y) = \beta_4(1.5Q^2 - 2.5Q^3 + Q^4) \ldots\ldots\ldots .62$$

$$Recall\ w = w(x) * w(y) = w(R) * w(Q)$$

$$w = \alpha_4\beta_4(R - 2R^3 + R^4) * (1.5Q^2 - 2.5Q^3 + Q^4)$$

$$Let \alpha_4\beta_4 = A$$

$$w_x = \sqrt{A}(R - 2R^3 + R^4) \ldots\ldots\ldots\ldots\ldots 63$$

$$w_Y = \sqrt{A}(1.5Q^2 - 2.5Q^3 + Q^4) \ldots\ldots\ldots\ldots .64$$

From equations (63) and (64), h_1 and h_2 are:

$$h_1 = R - 2R^3 + R^4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .65$$

$$h_2 = 1.5Q^2 - 2.5Q^3 + Q^4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots .66$$

Determination of the Stiffness Coefficients (ki) for the Two Plates with Various Boundary Conditions using Polynomial Functions for Both w_x and w_y:

The polynomial shape functions, h, of the two rectangular plates derived and recorded in equations 51,52,65&66 are used in the analysis for stiffness coefficients (k_x, k_{xy}, k_y and k_λ) of the two rectangular plates under study.

Recall the derived split-deflection resonating frequency equation for every thin rectangular plate based on work error method.

1. Edges 1 & 3 Clamped and Edges 2 and 4 Simply Supported (CSCS)

$$w = A\,(R - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)$$

$$w_x = \sqrt{A}(R - 2R^3 + R^4) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .67$$

$$w_y = \sqrt{A}(Q^2 - 2Q^3 + Q^4) \ldots\ldots\ldots\ldots\ldots\ldots 68$$

From equations (67) and (68), h_1 and h_2 are:

$$h_1 = (R - 2R^3 + R^4)$$

$$h_2 = (Q^2 - 2Q^3 + Q^4)$$

Integrating h_1^2 in a closed domain.

$$h_1^2 = (R - 2R^3 + R^4)(R - 2R^3 + R^4)$$

$$h_1^2 = R^2 - 2R^4 + R^5 - 2R^4 + 4R^6 - 2R^7 + R^5 + R^5 - 2R^7 + R^8$$

$$h_1^2 = R^2 - 4R^4 + 2R^5 + 4R^6 - 4R^7 + R^8$$

$$i.\ \int_0^1 h_1^2\, dR = \int_0^1 (R^2 - 4R^4 + 2R^5 + 4R^6 - 4R^7 + R^8)dR$$

$$\left(\frac{R^2}{3} - \frac{4R^5}{5} + \frac{2R^6}{6} + \frac{4R^7}{7} - \frac{4R^8}{8} + \frac{R^9}{9}\right)\begin{matrix}1\\0\end{matrix}$$

$$\frac{1}{3} - \frac{4}{5} + \frac{2}{6} + \frac{4}{7} - \frac{4}{8} + \frac{1}{9}$$

$$= \left(\frac{31}{630}\right)$$

Also integrating h_2^2 in a closed domain.

$$h_2^2 = (Q^2 - 2Q^3 + Q^4)(Q^2 - 2Q^3 + Q^4)$$

$$= Q^4 - 2Q^5 + Q^6 - 2Q^5 + 4Q^6 - 2Q^7 + Q^6 - 2Q^7 + Q^8$$

$$= Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8$$

ii. $\int_0^1 h_2^2 = (Q^4 - 4Q^5 + 6Q^6 - 4Q^7 + Q^8)dQ$

$$= \left(\frac{Q^5}{5} - 4\frac{Q^6}{6} + 6\frac{Q^7}{7} - 4\frac{Q^8}{8} + \frac{Q^9}{9}\right)\begin{matrix}1\\0\end{matrix}$$

$$= \left(\frac{1}{5} - \frac{4}{6} + \frac{6}{7} - \frac{4}{8} + \frac{1}{9}\right)$$

$$\left(\frac{1}{630}\right)$$

iii. $1st\ derivative\ of\ h_1 = d^1h_1 = 1 - 6R^2 + 4R^3$

$2nd\ derivative\ of\ h_1 = d^2h_1 = -12R + 12R^2$

$3rd\ derivative\ of\ h_1 = d^3h_1 = -12 + 24R$

$4th\ derivative\ of\ h_1 = d^4h_1 = 24$

(iv) $\int_0^1 \frac{d^2h_1}{dR^2} .h_1\, dR$

$$= \int_0^1 (-12R + 12R^2 +)(R - 2R^3 + R^4)\, dR$$

$$\int_0^1 \frac{d^2h_1}{dR^2} .h_1\, dR = \int_0^1 (-12R^2 + 24R^4 - 12R^5 + 12R^3 - 24R^5 + 12R^6)\, dR$$

$$\int_0^1 (-12R^2 + 12R^3 + 24R^4 - 36R^5 + 12R^6)\, dR$$

$$\left(-\frac{12R^3}{3} + \frac{12R^4}{4} + \frac{24R^5}{5} - \frac{36R^6}{6} + \frac{12R^7}{7}\right)\begin{matrix}1\\0\end{matrix}$$

$$\left(-\frac{12}{3} + \frac{12}{4} + \frac{24}{5} - \frac{36}{6} + \frac{12}{7}\right) = \left(-\frac{17}{35}\right)$$

(v) $\int_0^1 \frac{d^4h_1}{dR^4}.h_1\,dR = \int_0^1 24(R-2R^3+R^4)dR$

$\int_0^1 (24R-48R^3+24R^4)dR$

$\left(\frac{24R^2}{2}-\frac{48R^4}{4}+\frac{24R^5}{5}\right)\begin{matrix}1\\0\end{matrix}$

= 4.8

$Also, 1st\ derivative\ of\ h_2 = d^1h_2 = 2Q-6Q^2+4Q^3$

$2nd\ derivative\ of\ h_2 = d^2h_2 = 2-12Q+12Q^2$

$3rd\ derivative\ of\ h_2 = d^3h_2 = -12+24Q$

$4th\ derivative\ of\ h_2 = d^4h_2 = 24$

(vi) $\int_0^1 \frac{d^2h_2}{dQ^2}.h_2$

$=(Q^2-2Q^3+Q^4)(Q^2-2Q^3+Q^4)dQ$

$\int_0^1 (2Q^2-4Q^3+2Q^4-12Q^3+24Q^4-12Q^5+12Q^4-25Q^5+12Q^6)dQ$

$\int_0^1 (2Q^2-16Q^3+38Q^4-36Q^5+12Q^6)dQ$

$\left(\frac{2Q^3}{3}-\frac{16Q^4}{4}+\frac{38Q^5}{5}-\frac{36Q^6}{6}+\frac{12Q^7}{7}\right)\begin{matrix}1\\0\end{matrix}$

$\left(\frac{2}{3}-\frac{16}{4}+\frac{38}{5}-\frac{36}{6}+\frac{12}{7}\right)=-\frac{2}{105}$

(v) $\int_0^1 \frac{d^4h_1}{dR^4}.h_1\,dR = \int_0^1 24(R-2R^3+R^4)dR$

$\int_0^1 (24R-48R^3+24R^4)dR$

$\left(\frac{24R^2}{2}-\frac{48R^4}{4}+\frac{24R^5}{5}\right)\begin{matrix}1\\0\end{matrix}$

$=\frac{24}{2}-\frac{48}{4}+\frac{24}{5}=4.8$

(vi) $\int_0^1 \frac{d^4h_2}{dQ^4}.h_2\,dQ = \int_0^1 24(Q^2-Q^3+Q^4)\,dQ$

$\int_0^1 (24Q^2-48Q^3+24Q^4)\,dQ$

$$\left(\frac{24Q^3}{3} - \frac{48Q^4}{4} + \frac{24Q^5}{5}\right)\begin{matrix}1\\0\end{matrix}$$

$$\left(\frac{24}{3} - \frac{48}{4} + \frac{24}{5}\right) = \frac{4}{5}$$

Recalling equation (27) to (30) and substituting accordingly.

$$k_x = (4.8)\left(\frac{1}{630}\right) = \frac{4}{525} = 0.007619 \dots \dots .69$$

$$k_{xy} = \left(-\frac{17}{35}\right)\left(-\frac{2}{105}\right) = \frac{34}{3675} = 0.0092517 \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots .70$$

$$k_y = \left(\frac{31}{630}\right)\left(\frac{4}{5}\right) = \left(\frac{62}{1575}\right) = 0.039365 \dots \dots ..71$$

$$k_\lambda = \left(\frac{31}{630}\right)\left(\frac{1}{630}\right) = \left(\frac{31}{396{,}900}\right) = 0.000078105 \dots \dots \dots \dots \dots \dots .72$$

2.Edge 1 clamped and the other three edges simply supported (CSSS).

$$w_x = \sqrt{A}(R - 2R^3 + R^4) \dots \dots \dots \dots \dots \dots \dots .\ 73$$

$$w_y = \sqrt{A}(1.5Q^2 - 2.5Q^3 + Q^4) \dots \dots \dots \dots \dots .74$$

From equations (73) and (74), h_1 and h_2 are:

$$h_1 = (R - 2R^3 + R^4) \dots \dots \dots \dots \dots \dots \dots \dots \dots .75$$

$$h_2 = (1.5Q^2 - 2.5Q^3 + Q^4) \dots \dots \dots \dots \dots ..76$$

Integrating h_1^2 in a closed domain we would obtain:

$$(i) \int_0^1 h_1^2\, dR = \int_0^1 (R^2 - 2R^3 + R^4)^2\, dR$$

$$\int_0^1 h_1^2 = (R^2 - 2R^4 + R^5 - 2R^4 + 4R^6 - 2R^7 + R^5 - 2R^7 + R^8)dR$$

$$\int_0^1 h_1^2 = (R^2 - 4R^4 + 2R^5 + 4R^6 - 4R^7 + R^8)dR$$

$$\left(\frac{R^3}{3} - \frac{4R^5}{5} + \frac{2R^6}{6} + \frac{4R^7}{7} - \frac{4R^8}{8} + \frac{R^9}{9}\right)\begin{matrix}1\\0\end{matrix}$$

$$\left(\frac{1}{3} - \frac{4}{5} + \frac{2}{6} + \frac{4}{7} - \frac{4}{8} + \frac{1}{9}\right) \qquad = \frac{31}{630}$$

Integrating h_2^2 in a closed domain we shall have:

$$(ii) \int_0^1 h_2^2\, dQ = \int_0^1 (1.5Q^2 - 2.5Q^3 + Q^4)^2\, dQ$$

$$h_2^2 = (2.25Q^4 - 3.75Q^5 + 1.5Q^6 - 3.73Q^5 + 6.25Q^6 - 2.5Q^7 + 1.5Q^6 - 2.5Q^7 + Q^8)dQ$$

$$= \int_0^1 (2.25Q^4 - 7.5Q^5 + 9.25Q^6 - 5Q^7 + Q^8)\, dQ$$

$$= \left[\frac{2.25Q^5}{5} - \frac{7.5Q^6}{6} + \frac{9.25Q^7}{7} - \frac{5Q^8}{8} + \frac{Q^9}{9}\right]_0^1$$

$$= \left[\frac{2.25}{5} - \frac{7.5}{6} + \frac{9.25}{7} - \frac{5}{8} + \frac{1}{9}\right] = \quad \left(\frac{19}{2520}\right)$$

$1st\ derivative\ of\ h_1 = \ d^1h_1 = 1 - 6R^2 + 4R^3$

$2nd\ derivative\ of\ h_1 = d^2h_1 = -12R + 12R^2$

$3rd\ derivative\ of\ h_1 = d^3h_1 = -12 + 24R$

$4th\ derivative\ of\ h_1 = d^4h_1 = 24$

(iii) $\int_0^1 \frac{d^2h_1}{dR^2}\ .h_1\, dR.$

$$= \int_0^1 (-12R + 12R^2)(R^2 - 2R^3 + R^4)dR$$

$$\int_0^1 (-12R^2 + 24R^4 - 12R^5 + 12R^3 - 24R^5 + 12R^6)dR$$

$$\int_0^1 (-12R^2 + 12R^3 + 24R^4 - 36R^5 + 12R^6)dR$$

$$\int_0^1 \left(-\frac{12R^3}{3} + \frac{12R^4}{4} + \frac{24R^5}{5} - \frac{36R^6}{6} + \frac{12R^7}{7}\right)_0^1$$

$$= \left(-\frac{12}{3} + \frac{12}{4} + \frac{24}{5} - \frac{36}{6} + \frac{12}{7}\right) = \left(\frac{17}{35}\right)$$

$Also, 1st\ derivative\ of\ h_2 = \ d^1h_2$

$= 3Q - 7.5Q^2 + 4Q^3$

$2nd\ derivative\ of\ h_2 = d^2h_2 = 3 - 15Q + 12Q^2$

$3rd\ derivative\ of\ h_2 = d^3h_2 = -15 + 24Q$

$4th\ derivative\ of\ h_2 = d^4h_2 = 24$

(iv) $\int_0^1 \frac{d^2h_2}{dQ^2}\ .h_2\, dQ. = (3 - 15Q + 12Q^2)$

$((1.5Q^2 - 2.5Q^3 \quad + Q^4))dQ$

$$\frac{d^2h_2}{dQ^2} = (4.5Q^2 - 22.5Q^3 + 18Q^4 - 7.5Q^3 + 37.5Q^4 - 30Q^5 + 3Q^4 - 15Q^5 \\ + \quad 12Q^6)$$

$$\frac{d^2h_2}{dQ^2}.h_2 = (4.5Q^2 - 30Q^3 + 58.5Q^4 - 45Q^5 + 12Q^6)$$

$$\int_0^1 \frac{d^2h_2}{dQ^2}.h_2\,dQ = \int_0^1 (4.5Q^2 - 30Q^3 + 58.5Q^4 - 45Q^5 + 12Q^6)dQ$$

$$\left(\frac{4.5Q^3}{3} - \frac{30Q^4}{4} + \frac{58.5Q^5}{5} - \frac{45Q^6}{6} + \frac{12Q^7}{7}\right)_0^1$$

$$= \left(\frac{4.5}{3} - \frac{30}{4} + \frac{58.5}{5} - \frac{45}{6} + \frac{12}{7}\right) = \left(\frac{3}{35}\right)$$

(v) $\int_0^1 \frac{d^4h_1}{dR^4}.h_1\,dR = \int_0^1 24(R - 2R^3 + R^4)\,dR$

$$\int_0^1 (24R - 48R^3 + 24R^4)\,dR$$

$$\left(\frac{24R^2}{2} - \frac{48R^4}{4} + \frac{24R^5}{5}\right)_0^1$$

$$\frac{24}{2} - \frac{48}{4} + \frac{24}{5} \qquad = 4.8$$

(vi) $\int_0^1 \frac{d^4h_2}{dQ^4}.h_2\,dQ$

$$= \int_0^1 24\,(1.5Q^2 - 2.5Q^3 + Q^4)\,dQ$$

$$= \int_0^1 (36Q^2 - 60Q^3 + 24Q^4)\,dQ$$

$$\left(\frac{36Q^3}{3} - \frac{60Q^4}{4} + \frac{24Q^5}{5}\right)_0^1$$

$$= \left(\frac{36}{3} - \frac{60}{4} + \frac{24}{5}\right) \qquad = \left(\frac{9}{5}\right)$$

Recalling equatios (27) to (30) and substituting accordingly, we have

$$k_x = (4.8)\left(\frac{19}{2520}\right) = \frac{19}{525} = 0.0361905 \ldots\ 77$$

$$k_{xy} = \left(\frac{17}{35}\right)\left(\frac{3}{35}\right) = \left(\frac{51}{1225}\right) = 0.041632 \ldots 78$$

$$k_y = \left(\frac{31}{630}\right)\left(\frac{9}{5}\right) = \left(\frac{31}{350}\right) = 0.0885714 \ldots 79$$

$$k_\lambda = \left(\frac{31}{630}\right)\left(\frac{19}{2520}\right) = 0.000371 \ldots\ldots\ldots\ 80$$

PARTICULAR RESONATING FREQUENCY EQUATION FOR THE RECTANGULAR PLATES.

Substituting the values of the stiffness coefficients (k_x, k_{xy}, k_y& k_λ) in turn into equation (26), we obtained the particular resonating frequency equation for each rectangular plate.

Recall Equation. (26).

$$\lambda^2 = \frac{\left(k_x + \frac{2k_{xy}}{P^2} + \frac{k_y}{P^4}\right) * \frac{D}{ma^4}}{k_\lambda}$$

A. Edges 1 & 3 Clamped and Edges 2 and 4 Simply Supported (CSCS)

$$\lambda^2 = \left(\frac{\frac{4}{525} + \frac{68}{3675P^2} + \frac{62}{1575P^4}}{\frac{31}{396900}}\right) * \frac{D}{ma^4}$$

$$\therefore \lambda^2 = \left(97.548387 + \frac{236.90323}{P^2} + \frac{504}{P^4}\right) * \frac{D}{Ma^4} \ldots\ldots\ldots\ldots\ldots\ldots..81$$

B. Edge 1 Clamped and the other three Edges Simply Supported (CSSS)

$$\lambda^2 = \left(\frac{\frac{19}{525} + \frac{1}{P^2} \cdot \frac{102}{1225} + \frac{1}{P^4} \cdot \frac{31}{350}}{0.000371}\right) * \frac{D}{ma^4}$$

$$\therefore \lambda^2 = \left(97.548453 + \frac{231.035811}{P^2} + \frac{238.737004}{P^4}\right) * \frac{D}{ma^4} \ldots 82$$

RESULTS.

The non - dimensional form of the resonating frequencies for different aspect ratios for cscs and csss plates are shown on tables 1 and 2. Also, the non-dimensional resonating frequencies were plotted against the corresponding aspect ratios. The behaviour of the curves shows the relationship between the non-dimensional resonating frequencies of each rectangular plate and the corresponding aspect ratios.

Table 1: Non dimensional form of resonating frequency of cscs isotropic thin plate

Aspect ratio, P	Resonating frequency, $\lambda\left(\frac{1}{a^2}\sqrt{\frac{D}{m}}\right)$		Percentage difference
	Present	Past (Ibearugbulem et al., 2014)	
1	28.96	28.96	0.00
1.1	25.25	25.25	0.00
1.2	22.48	22.48	0.00
1.3	20.35	20.36	-0.05
1.4	18.70	18.70	0.00
1.5	17.39	17.39	0.00
1.6	16.33	17.34	-0.06
1.7	15.49	15.49	0.00
1.8	14.79	14.79	0.00
1.9	14.21	14.21	0.00
2	13.72	13.73	-0.07

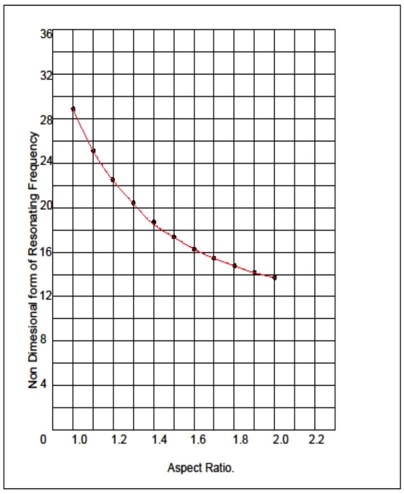

Fig.4.3. Non dimensional form of resonating frequency against aspect ratio (CSCS plate)

Table 2: Non dimensional form of resonating frequency of csss isotropic thin plate

Aspect ratio, P	Resonating frequency, $\lambda\left(\frac{1}{a^2}\sqrt{\frac{D}{m}}\right)$ Present	Past (Ibearugbulem et al., 2014)	Percentage difference
1	23.68	23.68	0.00
1.1	21.12	21.12	0.00
1.2	19.20	19.20	0.00
1.3	17.72	17.72	0.00
1.4	16.56	16.56	0.00
1.5	15.64	15.64	0.00
1.6	14.89	14.89	0.00
1.7	14.28	14.28	0.00
1.8	13.77	13.77	0.00
1.9	13.34	13.34	0.00
2	12.98	12.98	0.00

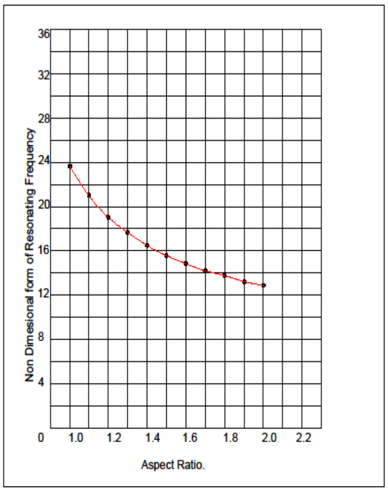

Fig.4.4. Non dimensional form of resonating frequency against aspect ratio (CSSS plate)

Considering Table 1 for the CSCS rectangular plate, the aspect ratio of 1.0 yielded the resonating frequency of 28.96. For the aspect ratio of 2.0 the resonating frequency was 13.72. Thus as the aspect ratio increased from 1.0 to 2.0, the resonating frequency decreased from 28.96 to 13.72. Therefore, for an increment of 1.0 (1.0-2.0) aspect ratio, there was a decrement of 15.24 (28.96-13.72) non-dimensional form of resonating frequency.

In figure 3, (CSCS plate), at the origin of the curve, the aspect ratio was 1.0 with non-dimensional resonating frequency of 23.68. The graph descended gradually to non-dimensional resonating frequency of 12.98. This corresponded to an aspect ratio of 2.0. At the beginning, the slope of the curve was very steep, while at the end,the slope reduced gradually.

In Table 2, the case of CSSS plates was considered. For the aspect ratio of 1.0, the resonating frequency was 23.68. For the aspect ratio of 2.0 the resonating frequency was 12.98. Thus as the aspect ratio increased from 1.0 to 2.0, the resonating frequency decreased from 23.68 to 12.98. Therefore, for an increment of 1.0 (1.0-2.0) aspect ratio, there was a decrement of 15.24 (28.96-13.72) non- dimensional form of resonating frequency

Considering the CSSS plate in Figure 4; at the origin of the curve, the aspect ratio was 1.0 with non-dimensional resonating frequency of 27.13. The graph descended gradually to non-dimensional resonating frequency of 17.84. This corresponded to an aspect ratio of 2.0. The implication of these are:

a. At the same aspect ratios, plates of different boundary conditions have different resonating frequencies.

b. For all the plates of different boundary conditions, as the aspect ratio increased, the resonating frequency decreased.

c. Different rectangular plates have different boundary conditions.

d. To design a structure with high resonating frequency, the aspect ratio must be made to be low.

CONCLUSIONS AND RECOMMENDATIONS.

A close examination of the tables reveals that the maximum percentage difference between the values from the present study and those from previous study, Ibearugbulem et al., 2014, is 0.07. From statistical point of view, this implies that no difference existed between the two sets of values. Thus, one can infer that the procedure, the deflection function and the energy function formulated in this present study are reliable and adequate in CPT free vibration analysis of rectangular plates. Hence, this method is recommended for stability analysis of CPT plates. It is also recommended that the present method is extended to refined plate theory analysis (RPT).

This small value of percentage difference from CSCS plate and zero value of percentage difference from the CSSS plate show that this present method is adequate and reliable for classical plate theory (CPT) free vibration analysis of rectangular plates

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